END TERM EXAMINATION
FIRST SEMESTER [B.TECH.] DEC 2013
APPLIED MATHEMATICS - I

Time: 3 Hours **M.M.:75**

NOTE: Attempt any five questions. Question no. 1 is compulsory. Attempt any one question from each section.

Question. 1.

(a) Find Maclaurin's series expansion of **sin *x*** using remainder theorem. **(3)**

(b) Find nth derivative of $\dfrac{x^3}{(x-1)(2x+3)}$. **(3)**

(c) Test the convergence of the series: $\sum_{n=1}^{\infty}\left(1-\cos\dfrac{\pi}{n}\right)$. **(3)**

(d) Using Caley-Hamilton theorem find A^8, if $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$. **(3)**

(e) Define Hermitian and Skew-Hermitian matrix with example. **(3)**

(f) Solve $\dfrac{d^2y}{dx^2}-y\tan x=y^4\sec x$. **(3)**

(g) Find the asymptotes of the curve $y^3-xy^2-x^2y+x^3+x^2-y^2=0$. **(3)**

(h) Evaluate $\int_0^{\pi} x\sin^7 x\cos^4 x\, dx$. **(3)**

(i) Express x^3+5x^2+6x+1 in terms of Legendre's polynomial. **(3)**

UNIT - I

Question. 2.

(a) Expand $\cos\left(m\sin^{-1}x\right)$ as far as x^6. **(6)**

(b) Discuss the convergence of the series: $\dfrac{x^2}{2\log 2}+\dfrac{x^3}{3\log 3}+\dfrac{x^4}{4\log 4}+\ldots$ **(6)**

Question. 3.

(a) Discuss the convergence and absolute convergence of the series: $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\ldots$; x being real. **(6)**

(b) Test the convergence of the series: $\sum_{n=1}^{\infty}\left(\dfrac{x^{n-1}}{1+x^n}\right); x>0$. **(6)**

UNIT - II

Question. 4.

(a) Trace the curve: $y^2(a+x)=x^2(3a-x)$. **(6)**

(b) Prove that $\Gamma m\,\Gamma\!\left(m+\dfrac{1}{2}\right)=\dfrac{\sqrt{\pi}\,\Gamma 2m}{2^{2m-1}}$; $m>0$. **(3)**

(c) Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}}dx = \frac{\pi}{4\sqrt{2}}$. **(3)**

Question. 5.

(a) Find the total length of the cardioids $r = a(1+\cos\theta)$ and show that its upper half is bisected by the line $\theta = \frac{\pi}{3}$. **(6)**

(b) Find the radius of curvature of the curve $y^2 = \frac{4a^2(2a-x)}{x}$ at a point where the curve meets the *x*-axis. **(6)**

UNIT - III

Question. 6.

(a) Find the values of λ for which the system of linear equations $3x_1 + x_2 - \lambda x_3 = 0$; $4x_1 - 2x_2 - 3x_3 = 0$ and $2\lambda x_1 + 4x_2 + \lambda x_3 = 0$ have non-trivial solution. For each permissible value of λ, find the general solution. **(6)**

(b) For the matrix $A = \begin{bmatrix} 2 & 1 & -3 & 6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$, find the non-singular matrices *P* and *Q* such that *PAQ* is in the normal form. Hence, find the rank of A. **(6)**

Question. 7.

(a) Reduce the quadratic form $2x^2 + 2y^2 + 3z^2 + 2xy - 4yz - 4xz$ to canonical form. Hence find their rank, index and signature. **(6)**

(b) Obtain eigen values and eigen vectors for the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ and verify that eigen vectors are orthogonal. **(6)**

UNIT - IV

Question. 8.

(a) State and prove orthogonality of Bessel's function. **(6)**

(b) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$ by the method of parameter of variation. **(6)**

Question. 9.

(a) Solve $(2xy\cos x^2 - 2xy + 1)dx + (\sin x^2 - x^2)dy = 0$. **(3)**

(b) Show that $\int_{-1}^{1} x P_n(x) P_{n-1}(x)\, dx = \frac{2n}{4n^2-1}$. **(3)**

(c) Solve the differential equation $(D^3 + 2D^2 + D)y = x^2e^{2x} + \sin^2 x$. $\left(D \equiv \frac{d}{dx}\right)$ **(6)**

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