MODEL TEST PAPER FOR END –TERM EXAMINATION

First Semester [B.Tech.] December 2013

APPLIED MATHEMATICS-I

Time: 3 Hours Maximum Marks: 75

Note: *Attempt five questions in all. Q. No. 1 is compulsory. Attempt one question from each unit.*

1. (a) Test for the convergence of the series $\sum \frac{n^{n^2}}{\left(n+\frac{1}{4}\right)^{n^2}}$ (3)

(b) Show that for any $x, \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + (-1)^n \frac{x^{2n}}{(2n)!}\sin\theta x$, $0 < \theta < 1$. (3)

(c) Find all the asymptotes of the curves: $y^2(x-2a) = x^3 - a^3$ (3)

(d) Trace the curves: $x^3 + y^3 = 3axy$ (3)

(e) Find the rank of matrix by Echlon form: $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ (3)

(f) Write down the quadratic forms corresponding to matrices: $\begin{bmatrix} 1 & 1 & -2 & 0 \\ 1 & -4 & 0 & 0 \\ -2 & 0 & 6 & -3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$ (3)

(g) Solve the differential equation: $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$ (3)

(h) Using Rodrigue's formula, prove the recurrence relation $P_n'(x) = xP_{n-1}'(x) + n\,P_n(x)$ (4)

Unit-1

Q2. (a) Find the n^{th} derivative of $\frac{1}{1+x+x^2}$. (6)

(b) Discuss the convergence of the following series: $\frac{x}{1} + \frac{1}{2}.\frac{x^3}{3} + \frac{1.3}{2.4}.\frac{x^5}{5} + \frac{1.3.5}{2.4.6}.\frac{x^7}{7} + \ldots$ $(6\frac{1}{2})$

Q3. (a) For what value of x are the following series convergent: $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \frac{x^5}{\sqrt{5}} - \cdots$ $(6\frac{1}{2})$

(b) Calculate the approximate value of $\sqrt{10}$ to four decimal places by taking the first four terms of an approximate Taylor's Series. (6)

Unit-2

Q4. (a) Prove that for ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\rho = \frac{a^2b^2}{p^3}$ where p is the perpendicular from the centre upon the tangent at (x, y). $(6\frac{1}{2})$

(b) Express the integral $\int_0^1 x^m (1-x^n)^p\,dx$ in terms of Gamma function and hence evaluate $\int_0^1 x^{3/2}\left(1-\sqrt{x}\right)^{1/2} dx$ (6)

Q5. (a) Determine the surface area of the solid generated when the leminiscate of Bernoulli

$r^2 = a^2 \cos 2\theta$ revolves about the initial line . $(6\frac{1}{2})$

(b) If $I_{m,n} = \int_0^{\pi/2} cos^m x \sin nx \, dx$. Show that $I_{m,n} = \frac{-\cos^m x \cos nx}{m+n} + \frac{m}{m+n} I_{m-1,n}$ (6)

Unit-3

Q6. (a) Use Gauss-Jordon Method to find the inverse of each of the matrices: $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ (6)

(b) Show that the system of equations $2x - 2y + z = \lambda x$, $2x - 3y + 2z = \lambda y$, $-x + 2y = \lambda z$ can possess a non-trivial solution only if $\lambda = 1$ or $\lambda = -3$. Obtain the general solution in each case.

$(6\frac{1}{2})$

Q7. (a) Let $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$, find a modal matrix P and corresponding spectral matrix D of A. $(6\frac{1}{2})$

(b) Verify Caley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Also express

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ as quadratic polynomial in A . (6)

Unit-4

Q8. (a) Apply the methods of variations of parameters to solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$ $(6\frac{1}{2})$

(b) Solve the following differential equations:

(i) $(D^2 - 3D + 2)y = 6e^{-3x} + \sin 2x$

(ii) $(D^2 - 1)y = \cos hx \cos x$ (3×2=6)

Q9. (a) Prove that the following recurrence relations of Bessel's function:

(i) $J_n'(x) = -J_{n+1}(x) + \frac{n}{x} J_n(x)$

(ii) $J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$ (3×2)

(b) Solve the differential equation: $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = \sin[2\log(1+x)]$ $(6\frac{1}{2})$