Assignment - 1

1. *(Venn Diagrams.)*
   (a) Draw the Venn diagram for $A \cup (B \Delta C)$.
   (b) Using as few symbols as possible, write down the sets corresponding to the shaded regions in the Venn diagrams in the Figure. For the first you should try to use 15 symbols including brackets ; for the second you should try to use at most 17 symbols including brackets.

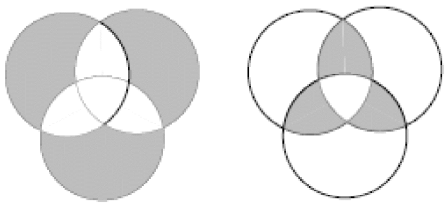

FIGURE 1 –

2. *(Set Identities.)* Prove the following set identity. In other words, show that if x is an element of the set on the left then it is also an element of the set on the right, and vice-versa.

$$(A \setminus B) \cup C = (A \cup C) \setminus (B \setminus C) .$$

3. *(Propositions.)* Which of the following sentences are propositions ?
   (a) Ouch.
   (b) There are finitely many primes.
   (c) $x^3 = 27$.
4. *(Conditional Statements.)* Which of the following implications are true.
   (a) If $1 + 1 = 3$ then $1 + 1 = 4$.
   (b) If 7 is prime then $2 + 3 = 5$.
   (c) If x is prime then $3x$ is prime.
5. *(Tautologies)* Which of the following are tautologies ? For the statements which *are* tautologies, give a proof. Use the rules of logic seen in class (stating at each step what rule you use), rather than using a truth table. For the statements which are *not* tautologies, give a counter-example, i.e., a truth assignment for which the proposition is false.
   (a) $p \vee [(\neg p) \vee q]$
   (b) $(p \vee q) \to p$
   (c) $(p \wedge q) \to p$
   (d) $[(p \to q) \to r] \to [(\neg p) \vee r]$
6. *(Logical Equivalence)* Verify the following logical equivalence using a truth table :

$$\neg(P \wedge (\neg Q \vee \neg R)) \leftrightarrow (\neg P) \vee (Q \wedge R) .$$

7. *(Circuits)* Suppose we flip four coins. Design a circuit to determine if an odd number come up "heads".