# Model Question Paper - 1

Sub : Applied Physics - II

Question no. 1 is compulsory. Attempt one question ~~each~~ from each unit.

Q.1 a) If E equals zero everywhere on the Gaussian surface, does Gauss's law require that there be no net charge inside?

b) Write Maxwell's equation in differential form.

c) In a plane electromagnetic wave in free space, the electric field oscillates at a frequency of $2 \times 10^{14}$ Hz. Find the wavelength of the wave.

d) Explain Wein's law and Rayleigh-Jeans law.

e) If an electron and a proton have the same kinetic energy, which has the shorter de Broglie wavelength?

f) Why is the energy of a particle trapped inside a box quantized? Give reasons.

g) Calculate the uncertainity in the velocity of an electron confined in a 10 Å box.

h) What are the basic lattice parameters?

i) Why X-rays are used to study crystal structure?

j) Why do energy bands arise in solids?

## Unit I

**Q 2(a)** State and Prove Poynting theorem. Explain the term Poynting vector. (8)

**Q 2(b)** The conduction current density $J$ in a dielectric is given as $J = 0.02 \sin(10^9 t)$ amp/$m^2$. Find the displacement current density if $\sigma = 10^3$ mho/m, $\varepsilon_r = 6.5$ and $\varepsilon_0 = 8.854 \times 10^{-12} c^2/N^2 m^2$ (4.5)

**Q 3(a)** Discuss the propagation of e.m waves in dielectric. Derive the expression for phase and wave impedence (8)

**Q 3(b)** In a plane electromagnetic wave the electric field oscillate at a frequency of $3 \times 10^{11}$ Hz and amplitude 50V/m. what is the wavelength of the wave. What is the amplitude of oscillating magnetic field. (4.5)

## Unit II

**Q. 4(a).** State and explain Heisenberg's Uncertainty principle. find the smallest possible uncertainty in position of the electron moving with velocity $3 \times 10^7$ m/Sec ($h = 6.63 \times 10^{-34}$ Js, $m_0 = 9.1 \times 10^{-31}$ kg) (6)

**(b)** Set up time independent schrödinger wave equation for a particle in 1-dimensinal box of size L. Obtain the wave functions and energy eigen values. (6.5)

**Q.5 (a)** Discuss and derive Planck's radiation formula. Compare analytically Planck's formula with Wien's and Rayleigh - Jean's Laws. (8)

**(b)** At what temperature can we expect a 10% probability that electrons in a metal will have an energy which is 1% above EF? Fermi energy = 5.5 ev (4.5)

Q.6(a) Describe the seven Crystal Structure Systems and their characteristics features.  (4)

(b) What is meant by unit cell? show that lattice constant 'a' for a cubic lattice is given by

$$a = \left(\frac{nM}{N f}\right)^{1/3}$$

(2.5)

(c) What are schottky and Frenkel defects? Derive an expression to (i) show that schottky defects (ii) in ionic crystals depend on tempt  (6)

OR

Q.7 (a) What are Miller indices? show that (i) a cubic crystal the spacing between consecutive parallel planes of Miller indices (h, k, l) is given by

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

(4.5)

(b) Derive the relation of Bragg's law for diffraction of X-rays  (4)

(c) Describe the NaCl and CsCl structure.  (4)

S.8 (a) Define effective mass and prove that effective mass of an electron $m^* = \dfrac{-h^2}{d^2E/dk^2}$.

(4)

(b) What is Hall effect? Show that Hall coefficient is independent of the applied magnetic field and is inversely proportional to the ~~current to velocity and~~ electronic charge

(6)

(c) Using free electron model, find the Hall coefficient of sodium assuming bcc structure for Na of cell side 4.28 $A°$

(25)

Ans. (a) The electrons in a crystal are not completely free but interact with crystal lattice. As a result, their behaviour towards external forces is different from that of a free electron. So we consider an altered value of mass called "Effective mass". Effective mass depends on the nature of crystal lattice and varies with the direction of motion of free $\bar{e}$ in the lattice.

Let us consider an $\bar{e}$ is moving along x-axis in a crystal in the presence of an external electric field E then it experiences a force $= eE$

work done = gain in Energy $\Rightarrow dE = F\,dx = eE\,dx$

$\therefore dE = eE\,v\,dt$ ———— (1)

we know that de-Broglie wave velocity

$v_g = \dfrac{dw}{dk}$ ———— (ii)

$$\therefore \quad v = vg$$

$$dE = eE \, vg \, dt \quad\text{------ (III)}$$

$$E = hv = \frac{h}{2\pi} \, \omega$$

$$dE = \frac{h}{2\pi} \, d\omega$$

$$= \frac{h}{2\pi} \frac{d\omega}{dK} \, dK$$

$$dE = \frac{h}{2\pi} \, vg \, dK \quad\text{------ (IV)}$$

from equ. (III) & (IV)

$$eE \, vg \, dt = \frac{h}{2\pi} \, vg \, dK$$

$$\frac{dK}{dt} = \frac{2\pi}{h} \, eE \quad\text{------ (V)}$$

$$\therefore \quad vg = \frac{d\omega}{dK} = \frac{2\pi}{h} \frac{dE}{dK}$$

$$\frac{dvg}{dt} = \frac{2\pi}{h} \frac{d^2E}{dK \, dt}$$

$$= \frac{2\pi}{h} \cdot \frac{d^2E}{(dK)^2} \cdot \frac{dK}{dt}$$

$$= \frac{2\pi}{h} \cdot \frac{d^2E}{dK^2} \cdot \frac{2\pi}{h} \cdot eE$$

$$\Big\{ vg = v$$

$$\frac{dv}{dt} = \left( \frac{4\pi^2}{h^2} \frac{d^2E}{dK^2} \right) eE$$

$$F = ma$$

$$a = F/m$$

$$\therefore \quad \frac{1}{m^*} = \frac{4\pi^2}{h^2} \cdot \frac{d^2E}{dK^2}$$

$$\boxed{m^* = h^2 \Big/ \frac{d^2E}{dK^2}}$$

(b) If a current carrying conductor is placed in an external magnetic field in direction perpendicular to the direction of flow of current, an electric field is produced in the wire in direction perpendicular to both the direction of current and direction of magnetic field. This effect is called "Hall effect."

Hall Coefficient:- Let the charge carriers in the rod be $\bar{e}s$ (charge = $-e$). Hall electric field $E_H$ (or $E_y$) will be in negative $Y$-direction.

force on electron due to Hall field $E_H = e E_H$ (+ve direction) & Lorentz force on $\bar{e}$ due to magnetic field

$$H_z = e(\vec{V} \times \vec{B})$$
$$= -e(-\hat{i} \, v_x \times \hat{K} \, H_z)$$
$$= e(\hat{i} \, v_x \times \hat{K} \, H_z)$$
$$= -\hat{j} \, e \, v_x H_z$$

Lorentz force $= e \, v_x H_z$ $(-ve \; Y$-direction $)$

In steady state $\quad e E_H = e \, v_x H_z$

$$E_H = v_x H_z \qquad — (1)$$

If 'd' is the thickness of the rod. Hall P.d.

$$V_H = E_H d = v_x H_z d \qquad — (11)$$

If $n \rightarrow$ no. of $\bar{e}s$ / unit volume in the rod

current density $\quad J_x = n(-e) v_x$

drift velocity $v_x = -\dfrac{J_x}{ne}$ ———— (III)

from (I) $\quad E_H = v_x H_z$

$$E_H = \dfrac{-J_x H_z}{ne}$$

$$E_H = -\dfrac{1}{ne}\,(J_x H_z)$$

$$E_H \propto J_x H_z$$

$$E_H = R_{Hall}\,(J_x H_z)$$

$$\boxed{R_{Hall} = -\dfrac{1}{ne}}$$

$$\boxed{R_{Hall} \propto \dfrac{1}{e}}$$

(e)   Given   $a = 4.28 \times 10^{-10}$ m.

unit cell of Na of volume $a^3$ has 2 atoms.

$$\therefore R_H = -\dfrac{1}{ne}$$

$$n = 2 \times \dfrac{1}{a^3} = \dfrac{2}{(4.28 \times 10^{-10})^3} = 2.55 \times 10^{28}/m^3$$

$$\therefore R_H = \dfrac{-1}{2.551 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= -0.245 \times 10^{-9}\ m^3/C.$$

**Q.9.** (a) Derive an expression for the electron concentration in an intrinsic semiconductor. **(6)**

(b) In an intrinsic semiconductor ($E_g = 0.676\,eV$), $m_e = 0.09\,m$ and $m_h = 0.36\,m$. Calculate the concentration of intrinsic charge carrier at $300K$. **(4)**

(c) Distinguish between intrinsic and extrinsic Semiconductors **(2.5)**

**Ans.** (a) <u>Electron Concentration in Conduction Band</u> :-

The density of e's in the conduction band ie total no. of e's per unit volume is given by

$$m_c = \int_{E_c}^{\infty} D(E)\, F(E)\, dE \quad \text{———(1)}$$

where $D(E) \rightarrow$ Energy density of states at the bottom of conduction band of the semiconductors is

$$D(E) = \frac{4\pi}{h^3}(2m_e)^{3/2}(E-E_c)^{1/2} \quad \text{———(11)}$$

$D(E)\,dE$ gives the total no. of available states in the range $E$ & $E + dE$

$F(E)$ is the fermi energy function giving the probability of occupation and given by

$$F(E) = \frac{1}{e^{(E-E_F/k_BT)}+1} \quad \text{———(111)}$$

where $k_B \rightarrow$ Boltzmann constant.

putting the values of $D(E)$ & $F(E)$ in equ. (1)

$$n_c = \frac{4\pi}{h^3}(2m_e)^{3/2} \int_{E_c}^{\infty} \frac{(E-E_c)^{1/2}\, dE}{e^{(E-E_f/K_BT)} + 1}$$

If $E - E_f \gg K_BT$, the unit term is denominator is negligible

$$n_c = \frac{4\pi}{h^3}(2m_e)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{(E_f-E)/K_BT}$$

$$n_c = \frac{4\pi}{h^3}(2m_e)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} \left\{ e^{\left(\frac{E_f-E_c}{K_BT}\right)} \cdot e^{\frac{E_c-E}{K_BT}} \right\} dE$$

Let $\frac{E-E_c}{K_BT} = x$

$$E - E_c = x K_BT$$

$$dE = K_BT\, dx$$

$$(E-E_c)^{1/2} = (x)^{1/2} (K_BT)^{1/2}$$

$$n_c = \frac{4\pi}{h^3}(2m_x)^{3/2} \int_0^{\infty} (x)^{1/2} (K_BT)^{1/2} e^{-x} e^{\left(\frac{E_f-E_c}{K_BT}\right)} \cdot K_BT\, dx$$

$$= \frac{4\pi}{h^3}(2m_e)^{3/2}(K_BT)^{3/2} e^{\left(\frac{E_f-E_c}{K_BT}\right)} \int_0^{\infty} e^{-x} x^{1/2}\, dx$$

$$\therefore \int_0^{\infty} x^{1/2} e^{-x}\, dx = (\pi/4)^{1/2}$$

$$n_c = \frac{4\pi}{h^3}(2m_e K_BT)^{3/2} e^{\left(\frac{E_f-E_c}{K_BT}\right)} \cdot (\pi/4)^{1/2}$$

$$\boxed{n_c = 2\left(\frac{2m_e K_B \pi T}{h^2}\right)^{3/2} \exp\left\{ \frac{E_f-E_c}{K_BT} \right\}} \quad -(iv)$$

(b.) $E_g = 0.676\ eV$ , $m_c = 0.09\ m$ , $m_h = 0.36\ m$

Then carrier concentration

$$n_i = 2\left(\frac{k_BT}{2\pi\hbar^2}\right)^{3/2}(m_c m_h)^{3/4}\exp\left(\frac{-E_g}{2k_BT}\right)$$

$$= C\ T^{3/2}(0.09\times0.36)^{3/4}\exp\left(-\frac{0.676}{0.052}\right)$$

$$C = 2\left(\frac{k_B}{2\pi\hbar^2}\right)^{3/2}$$

$$= 4.83\times10^{21}\ /\ m^3$$

$$T^{3/2} = (300)^{3/2} = 5.19\times10^{3}\ K^{3/2}$$

$$n_i = 4.83\times10^{21}\times5.19\times10^{3}\times0.0763\times$$

$$2.266\times10^{-6}$$

$$\boxed{n_i = 4.33\times10^{18}\ /\ m^3.}$$

(c)

| Intrinsic Semiconductor | Extrinsic Semiconductor |
|---|---|
| 1. A pure semiconductor | 1. Doped with impurities. |
| 2. Have low electrical conductivity | 2. Have high electrical conductivity. |
| 3. Operating temperature is low for this s/c | 3. Operating temperature is high. |
| 4. Charge carriers are produced due to thermal excitation | 4. Additional charge carriers are also produced due to added impurities |
| 5. Ex. — Ge, Si | 5. Ex. Ge & Si doped with P, As, Bi, Sb etc. |

# Model Question Paper - 1

## Solutions

**Q.1 a)** When $E$ is zero everywhere on the surface, then the net charge inside the Gaussian surface is always zero.

**b)** Maxwell's equations are

$$\text{div } \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\text{div } \vec{B} = 0$$

$$\text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{Curl } \vec{B} = \mu_0 \left[ \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right]$$

**c)** Wavelength, $\lambda = \frac{c}{2S}$ in free space, velocity of em wave in free space is $c$.

$$\lambda = \frac{3 \times 10^8}{2 \times 10^{14}} = 1.5 \, \mu m$$

**d)** Wien's displacement law states that the wavelength corresponding to maximum emission is inversely proportional to the absolute temperature.

$$\lambda_m T = \text{constant}$$

Wien's distribution law, the energy density of the blackbody radiation of wavelength between $\lambda$ and $(\lambda + d\lambda)$ is given by

$$E_\lambda \, d\lambda = \frac{A}{\lambda^5} e^{-B/\lambda T} \, d\lambda$$

Rayleigh - Jeans law

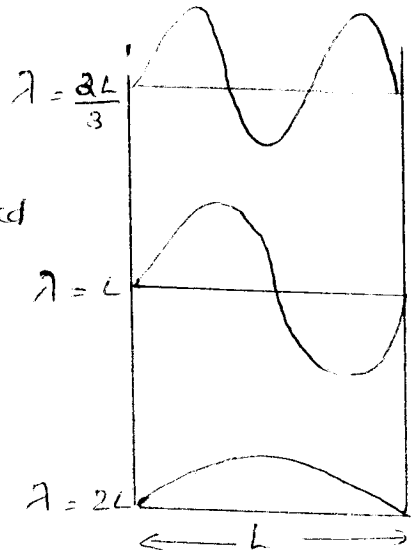$$E_\lambda \, d\lambda = \frac{8\pi}{\lambda^4} k_B T \, d\lambda$$

e)
$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$E_e = E_p = E$$

$$\lambda_e = \frac{h}{\sqrt{2m_e E}} \quad \text{and} \quad \lambda_p = \frac{h}{\sqrt{2m_p E}}$$

As $m_p > m_e$, $\lambda_p < \lambda_e$ ie the wavelength of proton is small compared to that of electron.

f)

Imagine a box of width $L$, and the possible de-Broglie wavelengths of a particle in a box are determined by the width $L$ of the box. The longest wavelength is $\lambda = 2L$, then next is $\lambda = L$, $\frac{2L}{3}$ ... or in general $\lambda = \frac{2L}{n}$ where $n = 1, 2, 3 \ldots$



$\lambda = \frac{2L}{3}$

$\lambda = L$

$\lambda = 2L$

$\longleftarrow L \longrightarrow$

Momentum $p = \frac{h}{\lambda}$

Kinetic energy $= \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

$$\lambda = \frac{2L}{n}$$

$$E = \frac{h^2 n^2}{2m(2L)^2} = \frac{n^2 h^2}{8mL^2}$$

where $n = 1, 2, 3 \ldots$ Thus the energy is quantized.

g)
$$\Delta x \, \Delta p = \frac{h}{4\pi}$$

$$\Delta p = \frac{h}{4\pi\Delta x} \quad \bullet \quad \Delta p = m\Delta v$$

$$\Delta v = \frac{h}{4\pi\Delta x \, m} = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 10 \times 10^{-10} \times 9.1 \times 10^{-31}}$$

$$= \frac{6.63 \times 10^{-34}}{114.3 \times 10^{-40}} = 0.058 \times 10^{6}$$

$$= 5.8 \times 10^{4} \, m/s$$

h) The basic lattice parameters determine the type and actual size of a unit cell. The intercepts a, b and c made by three adjacent faces with the crystallographic axes are called primitives or lattice constants. The angles $\alpha$, $\beta$ and $\gamma$ between the crystallographic axes are known as interfacial angles. The six quantities a, b, c and $\alpha$, $\beta$, $\gamma$ together form the lattice parameters.

i) In a crystal, atoms are arranged in a regular periodic manner and it can act as a three dimensional grating. To observe diffraction, the wavelength of the source must be comparable with the size of the obstacle. The typical interatomic spacing in a crystal is of the order of 1 Å and the wavelength of X-rays are also of the same range. That is why X-rays are used in crystal diffraction studies.

j) In solids atoms are very closely packed. If N number of atoms are brought together to form a solid, all the discrete energy levels of electrons of the constituent atoms come close together to form a continuous band of energies. Pauli's exclusion principle is not violated during the formation of an energy band.

Sol. of Q. 2(a)  Considered the medium as homogenous and isotropic, so we write one of the Maxwell's equation as

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\therefore \quad J = \nabla \times H - \frac{\partial D}{\partial t} \quad - (1)$$

The rate of doing work by the field in a given volume $V$ enclosed by surface $S$, is

$$\int_V (E \cdot v) dV = \int_V (E \cdot J) dV \quad - (2)$$

where $J = \rho v$

Putting ① in ② we get

$$\int_V (E \cdot J) dV = \int_V E \cdot \left(\nabla \times H - \frac{\partial D}{\partial t}\right) dV$$

$$= \int_V E \cdot (\nabla \times H) dV - \int_V \left(E \cdot \frac{\partial D}{\partial t}\right) dV$$

We use a vector identity

$$E \cdot (\nabla \times H) = \nabla \cdot (H \times E) + H \cdot (\nabla \times E)$$

Using $\nabla \times E = -\frac{\partial B}{\partial t}$, we get

$$E \cdot (\nabla \times H) = \nabla \cdot (H \times E) - H \cdot \frac{\partial B}{\partial t}$$

$$\int_V (E \cdot J) dV = \int_V \left[\nabla \cdot (H \times E) - H \cdot \frac{\partial B}{\partial t}\right] dV - \int_V \left(E \cdot \frac{\partial D}{\partial t}\right) dV$$

$$\int_V (E \cdot J) dV = \int_V \nabla \cdot (H \times E) dV - \int_V \left(E \cdot \frac{\partial D}{\partial t} + H \cdot \frac{\partial B}{\partial t}\right) dV$$

We have assumed the medium to be linear and isotropic, thus $D = \varepsilon E$ & $B = \mu H$

$$E \cdot \frac{\partial D}{\partial t} = E \cdot \frac{\partial}{\partial t}(\varepsilon E) = \frac{1}{2}\varepsilon \frac{\partial}{\partial t}(E)^2 = \frac{\partial}{\partial t}\left(\frac{1}{2} E \cdot D\right)$$

$$H \cdot \frac{\partial B}{\partial t} = H \cdot \frac{\partial}{\partial t}(\mu H) = \frac{1}{2}\frac{\partial}{\partial t}(H)^2 = \frac{\partial}{\partial t}\left(\frac{1}{2} H \cdot B\right)$$

$$\therefore \int_V (E \cdot J)dv = \int_V \nabla \cdot (H \times E)dv - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2} E \cdot D + \frac{1}{2} H \cdot B\right)dv$$

or

$$\int_V (E \cdot J)dv = -\int_V \nabla \cdot (E \times H)dv - \frac{\partial}{\partial t}\int_V \frac{1}{2}(E \cdot D + H \cdot B)dv$$

Using divergence theorem to change first volume integral on the right to the surface integral, we get

$$\int_V (E \cdot J)dv = -\int_S (E \times H) \cdot ds - \frac{\partial}{\partial t}\int_V \frac{1}{2}(E \cdot D + H \cdot B)dv$$

$$-\frac{\partial}{\partial t}\int_V \frac{1}{2}(E \cdot D + H \cdot B)dv = \int_V (E \cdot J)dv + \int_S (E \times H) \cdot ds \qquad \text{---(3)}$$

<u>Interpretation of each term</u> :

Since an electromagnetic field consist of electric and magnetic fields, thus the electrostatic energy in volume V is $u_e = \int \frac{1}{2}(E \cdot D)dv$ and $u_m = \int \frac{1}{2}(H \cdot B)dv$ is the magnetic energy in volume V. Thus the total energy of electromagnetic field is

$$U = u_e + u_m = \int \frac{1}{2}(E \cdot D + B \cdot H)dv$$

Thus the integral on the left hand of eq (3) may be interpreted as the rate of decrease of electromagnetic energy stored in volume V. The first term on the right side gives the rate of doing work by the field forces on the charges contained in the volume.

**3(a)**

In a dielectric medium, $\rho = 0 \quad \sigma = 0 \quad J = 0$

So the Maxwell's equation take the form

$$\nabla \cdot D = 0 \quad - (1)$$

$$\nabla \cdot B = 0 \quad - (2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad - (3) \quad \text{where} \quad D = \varepsilon E$$

$$B = \mu H$$

$$\nabla \times H = \frac{\partial D}{\partial t} \quad - (4)$$

Taking curl of eq (3)

$$\nabla \times \nabla \times E = -\nabla \times \left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times H)$$

$$= -\frac{\partial}{\partial t}(\mu(\nabla \times H)) \qquad \text{using eq (4)}$$

$$= -\mu \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right)$$

$$\nabla \times \nabla \times E = -\mu \varepsilon \frac{\partial^2 E}{\partial t^2} \qquad - (5)$$

Using vector identity

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E$$

where $(\nabla \cdot E) = 0$

So $\qquad \nabla \times \nabla \times E = -\nabla^2 E \qquad --- (6)$

from (5) & (6) we get

$$\nabla^2 E = \varepsilon \mu \frac{\partial^2 E}{\partial t^2}$$

Similarly $\quad \nabla^2 H = \varepsilon \mu \frac{\partial^2 H}{\partial t^2} \qquad - (7)$

(6) & (7) equation are familiar with a general equation of an undamped wave

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \qquad (8)$$

The second term represents the energy that flow out of boundary per unit time.

Poynting vector :- Poynting vector is defined as the flow of electromagnetic energy per unit area per unit time across the boundary. It is denoted by the symbol N

$$N = E \times H$$

its unit is watt/m²

Ans 2(b)

$$J = \sigma E$$
$$E = J/\sigma$$

$$J = 0.02 \sin(10^9 t)$$
$$\sigma = 10^3 \, mho/m$$
$$E = \frac{0.02 \sin(10^9 t)}{10^3} = 2 \times 10^{-5} \sin(10^9 t) \, V/m$$

The displacement current density is given as

$$\frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t} = \epsilon \epsilon_r \frac{\partial}{\partial t} \left( 2 \times 10^{-5} \sin(10^9 t) \right)$$

$$\epsilon_r = 6.5 \quad \& \quad \epsilon_0 = 8.85 \times 10^{-12} \, C^2/N\text{-}m^2$$

$$\frac{\partial D}{\partial t} = 1.151 \times 10^{-6} \cos(10^9 t) \, amp/m^2$$

So comparing (6) & (7) with (8)

$$v = \frac{1}{\sqrt{\varepsilon\mu}} \quad \text{or} \quad = \quad v = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}} \quad = \quad \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

is represents the velocity of light in dielectric medium, so the velocity of ~~light is les~~ em wave in dielectric medium is less that velocity of em wave in free space

The wave equations for the electromagnetic waves in dielectric medium are

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2 H - \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} = 0$$

So solution of the above eq

$$E = E_0 e^{i(K\cdot r - \omega t)}$$

$$H = H_0 e^{i(K\cdot r - \omega t)}$$

Phase between E & H

$$\text{Curl } E = -\mu \frac{\partial H}{\partial t}$$

on solving we get

$$i K \times E = -\mu(-i\omega H)$$

or

$$K \times E = \mu \omega H$$

Phase between E & H

$$H = \frac{1}{\mu\omega}(K \times E)$$

$$= \frac{K}{\mu\omega}(\hat{n} \times E)$$

$$= \frac{1}{\mu c}(\hat{n} \times E)$$

This equation in term of modulus gives

$$H = \frac{1}{\mu c} E$$

the ratio of E & H is represented by Z

$$\boxed{Z = \frac{E}{H} = \mu c}$$

Z is referred to as the wave impedence of free space.

3(b)

$$c = n\lambda$$
$$\lambda = c/n$$
$$c = 3 \times 10^8 \, m/s \qquad n = 3 \times 10^{11} \, H_z$$
$$\lambda = \frac{3 \times 10^8}{3 \times 10^{11}} = 10^{-3} \, m$$

further $\frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}}$ & $B_0 = \mu_0 H_0$

$$\frac{E_0}{B_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{1}{\mu_0} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$$

$$B_0 = \frac{E_0}{c} = \frac{50}{3 \times 10^8} = 1.6 \times 10^{-7} \, Tesla$$

Ans. 5(a)

# Planck's Radiation Law →

According to the classical theory of radiation, energy changes of radiators take place continuously. The classical theory failed to explain the experimentally observed distribution of energy in the spectrum of black body.

Plank succeeded in deriving a formula which agrees extremely well with experimental results. He discarded both the idea of radiation being a continuous stream as well as the law of equipartition of energy. He suggested the quantum theory of radiation. His assumptions are:

A black-body radiation chamber is filled up not only with radiation, but also with simple harmonic oscillators or resonators of molecular dimensions. They can vibrate with all possible frequencies.

The oscillators or resonators cannot radiate or absorb energy continuously. But an oscillator of frequency $\nu$ can only radiate or absorb energy in units of quanta of magnitude $h\nu$. $h$ is a universal constant called Plank's constant. The emission of radiation corresponds to a decrease and absorption to an increase in the energy and amplitude of an oscillator.

## Derivation →

Let $N$ be the total number of Planck's resonators and $E$ their total energy.

Then average energy per oscillator $= \bar{E} = \dfrac{E}{N}$ ____ (1)

If $N_0, N_1, N_2 \ldots N_r$ etc. are the number of oscillators having energies $0, \varepsilon, 2\varepsilon \ldots r\varepsilon$ etc. ____ (2)

then $N = N_0 + N_1 + N_2 \ldots N_r + \cdots$ ____ (3)

$E = 0 + \varepsilon N_1 + 2\varepsilon N_2 \ldots r\varepsilon N_r$

from Maxwell's distribution law it follows that the number of oscillators having energy $r\varepsilon$ will be

$$N_r = N_0 e^{-r\varepsilon/KT}$$

$$\Rightarrow \quad N_1 = N_0 e^{-\varepsilon/KT} \qquad (\text{Number of resonators having energy } \varepsilon)$$

$$N_2 = N_0 e^{-2\varepsilon/KT} \qquad (\text{Number of } \dots \dots \dots \quad 2\varepsilon)$$

Therefore

$$N = N_0 + N_0 e^{-\varepsilon/KT} + N_0 e^{-2\varepsilon/KT} + \dots N_0 e^{-r\varepsilon/KT} \dots \qquad ④$$

Putting $\varepsilon/KT = x$

$$N = N_0 + N_0 e^{-x} + N_0 e^{-2x} \dots \dots N_0 e^{-rx} \dots$$

$$\therefore \quad N = N_0 \left[ 1 + e^{-x} + e^{-2x} \dots \dots + e^{-rx} \dots \right]$$

$$N = \frac{N_0}{1 - e^{-x}} \qquad ⑤$$

The total energy of Planck's resonator is

$$E = 0 \times N_0 + \varepsilon \times N_0 e^{-x} + 2\varepsilon \times N_0 e^{-2x} \dots \dots r\varepsilon \times N_0 e^{-rx} + \dots$$

$$= N_0 \varepsilon \left[ e^{-x} + 2 e^{-2x} + \dots \dots r e^{-rx} \right]$$

$$= N_0 \varepsilon\, e^{-x} \left[ 1 + 2 e^{-x} + 3 e^{-2x} + \dots \dots r e^{-(r-1)x} \dots \right]$$

$$E = \frac{N_0 \varepsilon\, e^{-x}}{(1 - e^{-x})^2} \qquad ⑥ \qquad\qquad \left\{ \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \dots \right\}$$

$$\text{Average energy} \atop \text{of a resonator} \left. \right\} = \bar{\varepsilon} = \frac{E}{N} = \frac{\varepsilon\, e^{-x}}{1 - e^{-x}} = \frac{\varepsilon}{e^{x} - 1} \qquad ⑥$$

According to Planck's hypothesis

$$\varepsilon = h\nu$$

or

$$\varepsilon = \frac{hc}{\lambda}$$

and

$$x = \frac{\varepsilon}{KT} = \frac{hc}{\lambda KT} \qquad ⑦$$

$\therefore$ from (6)

$$\Rightarrow \quad \bar{\varepsilon} = \frac{\hbar c/\lambda}{\left(e^{\hbar c/\lambda kT} - 1\right)} \qquad \underline{\qquad} \quad \text{⑧}$$

in Cave

Number of oscillators per unit volume in the wavelength range $\lambda$ and $\lambda + d\lambda$

$$= \frac{8\pi \nu^2}{c^3} \, d\nu \qquad \text{or} \quad = \frac{8\pi}{\lambda^4} \, d\lambda \qquad \underline{\qquad} \quad \text{⑨}$$

Hence, energy density of radiation between wavelengths $\lambda$ and $\lambda + d\lambda$ = (average energy of a Planck's oscillator) × (number of oscillators per unit volume)

$$E_\lambda \, d\lambda \quad = \quad \frac{\hbar c/\lambda}{\left(e^{\hbar c/\lambda kT} - 1\right)} \quad \times \quad 8\pi \lambda^{-4} \, d\lambda \quad = \quad \frac{8\pi \hbar c \, \lambda^{-5}}{\left(e^{\hbar c/\lambda kT} - 1\right)} \, d\lambda$$

$$\underline{\qquad} \quad \text{⑩}$$

or

$$E_\nu \, d\nu = \frac{8\pi \hbar \nu^3}{c^3 \left(e^{\hbar \nu/kT} - 1\right)} \cdot d\nu \qquad \underline{\qquad} \quad \text{⑪}$$

Hence $E_\nu \, d\nu$ is the energy density belonging to the range $d\nu$. above equations represent Planck's radiation Law in terms of wavelength.

## Derivation of various Laws from Planck's Law —

**1. Wien's Law →**

Planck's radiation law is

$$E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda KT} - 1} \, d\lambda \qquad ---(1)$$

For small temperatures $\lambda T$ is small, and for shorter wavelengths $e^{hc/\lambda KT}$ becomes larger as compared to unity and hence, Planck's Law reduces to

$$E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda KT}} \, d\lambda$$

$$\boxed{E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \, e^{-hc/\lambda KT} \, d\lambda} \qquad ---(2)$$

Wien's Law

**2. Rayleigh - Jean's Law →**

For large temperature, $\lambda T$ is large and also for longer wavelength $e^{hc/\lambda KT}$ may be approximated to $\left(1 + \frac{hc}{\lambda KT}\right)$ as follows:

$$e^{hc/\lambda KT} = 1 + \frac{hc}{\lambda KT} + \left(\frac{hc}{\lambda KT}\right)^2 \frac{1}{2!} + \cdots$$

$$= 1 + \frac{hc}{\lambda KT} \qquad ---(3)$$

putting these values in eq$^n$ (1)

$$E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\left[1 + \frac{hc}{\lambda KT} - 1\right]} = \frac{8\pi hc}{\lambda^5} \frac{(\lambda KT)}{hc} \cdot d\lambda$$

$$\boxed{E_\lambda \, d\lambda = \frac{8\pi KT}{\lambda^4} \cdot d\lambda} \qquad ---(4)$$

Rayleigh Jean's law.

5(b)        $F(E) = 10\%$

$E = E_f + 1\%$ of $E_f$   ,  $E_f = 5.5$ eV,  $T = ?$

$F(E) = \dfrac{1}{\left(e^{(E-E_f)kT} + 1\right)}$

$E = 5.5 + \dfrac{5.5}{100} = 5.5 + 0.555$

$E - E_f = 0.555$

$\Rightarrow$    $0.1 = \left[\dfrac{1}{\exp \dfrac{0.555 \times 1.6 \times 10^{19}}{1.38 \times 10^{23}\, T}} + 1\right]$

$0.1 = \dfrac{1}{\exp\left(\dfrac{637.7}{T}\right) + 1}$

$\underline{T = 290.2\ K}$

3.6(a) Describe the seven crystal structure systems and their characteristics features. (4)

Ans:

| Crystal system | Relationship in the sides of unit cell | Relationship in the angles b/w the axes of unit cell | Examples | No. of possible lattices |
|---|---|---|---|---|
| Cubic | $a=b=c$ | $\alpha=\beta=\gamma=90°$ | NaCl, $CaF_2$ | Scc, bcc, fcc |
| Tetragonal | $a=b\neq c$ | $\alpha=\beta=\gamma=90°$ | $NiSO_4$, $SnO_2$ | Scc, bcc |
| Ortho-hombic | $a\neq b\neq c$ | $\alpha=\beta=\gamma=90°$ | $KNO_3$, $BaSO_4$ | Scc, bcc, fcc, base. |
| Mono-clinic | $a\neq b\neq c$ | $\alpha=\beta=90°\neq\gamma$ | $Na_2SO_4$, $FeSO_4$ | Scc, base |
| Triclinic | $a\neq b\neq c$ | $\alpha\neq\beta\neq\gamma\neq90°$ | $CuSO_4$, $K_2Cr_2O_7$ | Scc |
| Trigonal | $a=b=c$ | $\alpha=\beta=\gamma\neq90°$ | $CaSO_4$, As, Sb | Scc |
| Hexagonal | $a=b\neq c$ | $\alpha=\beta=90°, \gamma=120°$ | $SiO_2$, AgI, ZnCd | Scc |

3.6(b) What is meant by unit cell ?. Show that lattice constant 'a' for a cubic lattice is given by (2.5)

$$a = \left(\frac{nM}{N\rho}\right)^{1/3}$$

Ans. Unit Cell → A unit cell of a crystal can be assumed to be the solid volume, by the three dimensional repetition of which crystal is obtained. A crystal is composed of all identical unit cells.

lattice Constant for a Cubic lattice →

Let a be the side of the unit cell of a cubica
lattice.

∴ Volume of unit Cell $V = a^3$

If $\rho$ → density of Material of crystal

∴ mass of unit cell $= a^3 \rho$  — ①

Let n → no. of lattice points per unit cell

M → molecules wt of the crystal

N → Avogadro's no.

then mass of each molecule $m = \dfrac{M}{N}$

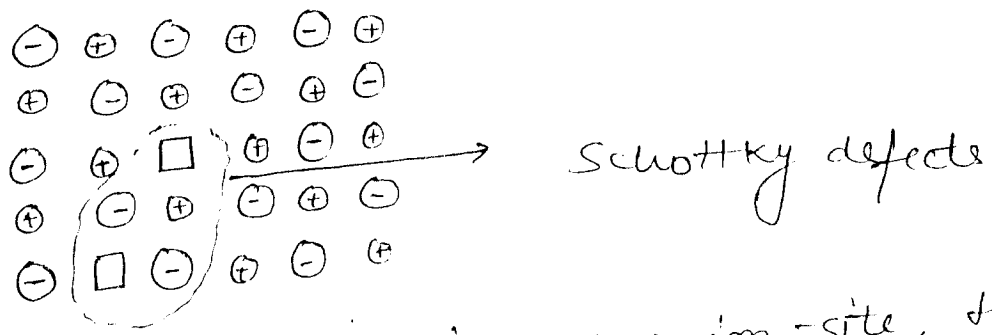∴ Mass of each unit cell $= n \times m = \dfrac{nM}{N}$  — ②

from eq. ① & ②

$$a^3 \rho = \dfrac{nM}{N}$$

$$\boxed{\text{Lattice Constant} \quad a = \left(\dfrac{nM}{\rho N}\right)^{1/3}}$$

Q·6(C) what are schottky and Frenkel defects?
Derive an expression to show that schottky
defects in ionic crystals depend on temperature. [6]

Ans: schottky defects → A pair of one cation and one
anion can be missing from an ionic crystal.
Such a pair of vacant ion sites is called
schottky defects.

 → Schottky defects

If there is a vacancy in a +ve ion-site, the charge neutrality may be achieved by creating vacancy in neighbouring negative ion site. Such a pair of vacant sites is called Schottky defects.

Eg. NaCl, KCl (Alkali halides).

Frenkel defects → When an atom is transferred from the regular site to an interstitial Position (void space between atoms, i.e; the Position not normally occupied by an atom) is called as Frenkel defect. Eg. AgCl, AgBr (Silver halide).

 Frenkel defects
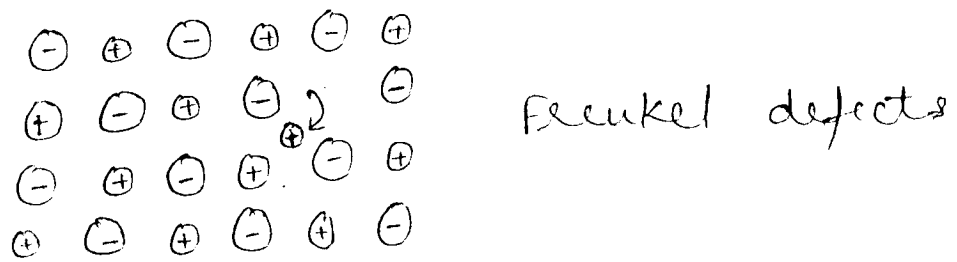
Concentration of Schottky defect in a crystal at tempt. T →

Consider a pure crystal made up of equal no. of oppositely charged ions.

Let:

$n \rightarrow$ no. of schottky defects (no. of vacancy pairs creat A-)

$N \rightarrow$ total no. of ions in the crystal

$E_p \rightarrow$ energy required to creat a pair of vacancy.

$\therefore nE_p \rightarrow$ rse in energy associated with the vacant sites.

$\therefore$ total no. of ways in which an anion or cation vacancy may be created $= \dfrac{N!}{n!(N-n)!}$

Therefore, the total no. of ways in which $n$ pairs of cation - anion vacancies may be created as

$$W = \left[\frac{N!}{n!(N-n)!}\right]\left[\frac{N!}{n!(N-n)!}\right] = \left[\frac{N!}{n!(N-n)!}\right]^2$$

as disorderness rses, entropy rse.

$$S = K \log W = K \log \left[\frac{N!}{n!(N-n)!}\right]^2$$

$\therefore$ change in helmholtz free energy

$$F = U - TS$$
$$= nE_p - TK \log \left[\frac{N!}{n!(N-n)!}\right]^2$$
$$= nE_p - 2kT\left[\log N! - \log n! - \log(N-n)!\right]$$

using stirling's formula

$$\log x! = x \log x - x$$

$$\therefore F = nE_p - 2kT\left[N\log N - \cancel{N} - n\log n + \cancel{n} - (N-n)\log(N-n) + (\cancel{N}-\cancel{n})\right]$$

$$F = nE_p - 2kT\left[N\log N - n\log n - (N-n)\log(N-n)\right] \quad \text{---(1)}$$

a) At thermal equilibrium, free energy is const.

$$\frac{\partial F}{\partial n}\Big|_T = 0$$

∴ Diff eq. ① w.r.to n

$$\frac{\partial F}{\partial n} = E_p - 2KT \left[0 - \left\{\frac{n}{n} + \log n\right\} - \left\{\frac{N-n}{(N-n)}(-1) + (-1)\log(N-n)\right\}\right]$$

$$= E_p - 2KT \left[-1 - \log n + 1 + \log(N-n)\right]$$

$$= E_p - 2KT \left[\log\left(\frac{N-n}{n}\right)\right]$$

Put $\frac{\partial F}{\partial n} = 0$

$$\therefore E_p - 2KT \left[\log\left(\frac{N-n}{n}\right)\right] = 0$$

$$\frac{E_p}{2KT} = \log\left(\frac{N-n}{n}\right)] \qquad \Rightarrow \quad e^{E_p/2KT} = \frac{N-n}{n}$$

∵ N >> n ( no. of vaccancy pairs is << total
no. of ions in the crystal )

∴ N - n ≈ N

$$\frac{N}{n} = e^{E_p/2KT} \qquad \text{or} \qquad \boxed{n = N e^{-E_p/2KT}}$$

⇒ no. of defects (vacancy pairs) ɪɪɪ ɪɪɪ
exponentially with temp.

Q7. What are Miller indices ?. Show that in a cubic crystal the spacing b/w consecutive parallel planes of Miller indices (h, k, l) is given by          (4.5)

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Ans: Miller indices are the smallest integers whose ratio is equal to the reciprocal of the intercept on the x, y & z axes axes by the plane.

If the lengths of intercepts on the x, y, & z axes by a crystal plane are respt & $l_1, l_2$ & $l_3$ then

$$h : k : l = \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$$

spacing b/w the lattice planes →

Fig (a) shows a 2-D lattice. Let a lattice plane AB cuts intercepts 2a & 3b on the OX and OY axes respt (OA = 2a & OB = 3b). The Miller indices of this plane will be (3 2)
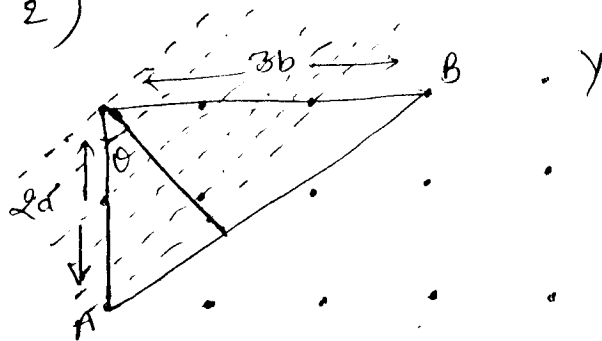


fig (a)

Let a normal OC be drawn from the point O on the plane AB. from fig, it is clear that

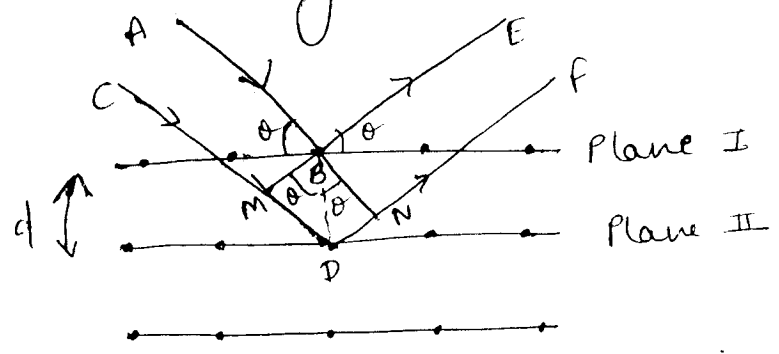Q.7(b) Derive relation of Bragg's law for diffraction of X-rays.
(4)

Ans: According to Bragg's law, if d is the seperation b/w the 2 nearest planes & θ is the glancing angle of the incident X-rays with the crystal plane then for Constructive diffraction

$$2d \sin\theta = n\lambda$$

where $n = 1, 2, 3 \ldots$ an integer.

Consider 2 rays AB & CD incident on atoms at B & D & reflected along BE & DF.



Draw $\perp^{ar}$ from B to CD & DF. The Path diff. b/w the 2 rays is MD + DN.

From right angled triangles

$$\sin\theta = \frac{MD}{BD} = \frac{DN}{BD}$$

$$\sin\theta = \frac{MD}{d} = \frac{DN}{d}$$

$$MN = DN = d\sin\theta$$

∴ Path diff = MD + DN = $d\sin\theta + d\sin\theta = 2d\sin\theta$

For Max^m intensity, Path diff must be equal to an even multiple of $\lambda/2$ i.e.,

Path diff. = $n\lambda$

$$\Rightarrow \boxed{2d\sin\theta = n\lambda}$$

Hence proved.
This is Bragg's law.

the $\triangle AOC$ & $\triangle OBA$ are similar. Hence,

$$\frac{OC}{OA} = \frac{OB}{AB} \quad \text{or} \quad OC = \frac{OB \times OA}{AB}$$

$$OC = \frac{(3b) \times (2a)}{\sqrt{(3b)^2 + (2a)^2}} = \frac{1}{\sqrt{\frac{(3b)^2 + (2a)^2}{(3b)^2 (2a)^2}}} = \frac{1}{\sqrt{\left(\frac{1}{2a}\right)^2 + \left(\frac{1}{3b}\right)^2}}$$

Since there are 6 parallel and equidistant crystal planes b/w O & AB which are shown by the dotted ~~two~~ lines, therefore separation b/w the 2 consecutive crystal planes

$$d = \frac{OC}{6} = \frac{1}{\sqrt{\left(\frac{6}{2a}\right)^2 + \left(\frac{6}{3b}\right)^2}} = \frac{1}{\sqrt{\left(\frac{3}{a}\right)^2 + \left(\frac{2}{b}\right)^2}}$$

$$= \frac{1}{\sqrt{\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2}} = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2}}}$$

∴ Since Miller indices of the crystal plane AB are 3, 2 hence $h = 3$ & $k = 2$.

Similarly, in a 3-D lattice (or crystal), the distance b/w Consecutive crystal planes is given b as

$$d_{hkl} = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}} = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{-1/2}$$

For cubic crystal $a = b = c$

$$\boxed{d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}}$$

Q 7 (c) Describe the NaCl & CsCl structures. (4)

Ans Fig shows the NaCl structure. The lattice is fcc. in which there is a basis of 2 atoms Na+ Cl at the each lattice point.

The no. of Molecules per unit cell is 4. The coordinates of sodium atoms are $(0,0,0)$, $(1/2, 1/2, 0)$, $(1/2, 0, 1/2)$, $(0, 1/2, 1/2)$ & the coordinates of chlorine atoms are $(1/2, 1/2, 1/2)$, $(0, 0, 1/2)$, $(0, 1/2, 0)$, $(1/2, 0, 0)$. The no. of nearest unlike atoms is 6.

Eg. NaCl, PbS
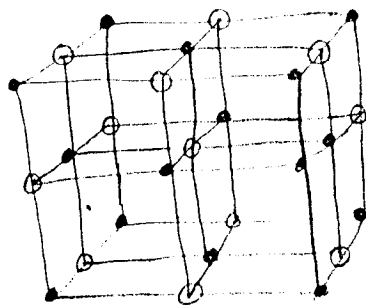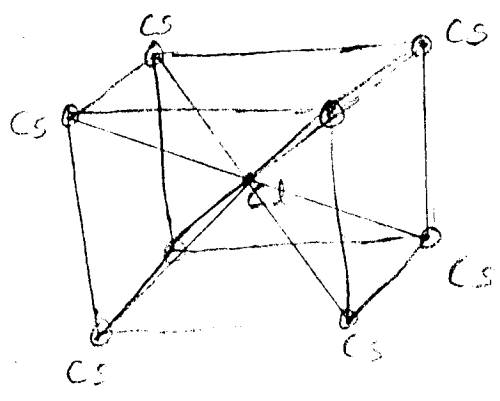


• Na
○ Cl

NaCl structure

CsCl Structure →

Fig shows the cesium chloride structure. In this structure the lattice is body centered lattice in which there are like atoms at each Corners of the unit cell & an unlike atom at the centre of the cell. i.e; at each of the eight corners of the cell there is Cl atoms & at the ~~eight corner of the~~ centre of the cell there is Cs atom. The coordinates of Cs atom are $(0,0,0)$ & the coordinates

. of chlorine atom are $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$

. In each unit cell, there is one atom of one kind and one atom of other kind i.e; no. of molecules per unit cell is 1.

Eg., CsCl, RbCl



CsCl structure